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The California Mathematics Content Standards

A high-quality mathematics program is essential for all students and provides every student with the opportunity to choose among the full range of future career paths. Mathematics, when taught well, is a subject of beauty and elegance, exciting in its logic and coherence. It trains the mind to be analytic—providing the foundation for intelligent and precise thinking.

To compete successfully in the worldwide economy, today's students must have a high degree of comprehension in mathematics. For too long schools have suffered from the notion that success in mathematics is the province of a talented few. Instead, a new expectation is needed: all students will attain California's mathematics academic content standards, and many will be inspired to achieve far beyond the minimum standards.

These content standards establish what every student in California can and needs to learn in mathematics. They are comparable to the standards of the most academically demanding nations, including Japan and Singapore—two high-performing countries in the Third International Mathematics and Science Study (TIMSS). Mathematics is critical for all students, not only those who will have careers that demand advanced mathematical preparation but all citizens who will be living in the twenty-first century. These standards are based on the premise that all students are capable of learning rigorous mathematics and learning it well, and all are capable of learning far more than is currently expected. Proficiency in most of mathematics is not an innate characteristic; it is achieved through persistence, effort, and practice on the part of students and rigorous and effective instruction on the part of teachers. Parents and teachers must provide support and encouragement.

These content standards establish what every student in California can and needs to learn in mathematics.

The standards focus on essential content for all students and prepare students for the study of advanced mathematics, science and technical careers, and post-secondary study in all content areas. All students are required to grapple with solving problems; develop abstract, analytic thinking skills; learn to deal effectively and comfortably with variables and equations; and use mathematical notation effectively to model situations. The goal in mathematics education is for students to:

- Develop fluency in basic computational skills.
- Develop an understanding of mathematical concepts.
- Become mathematical problem solvers who can recognize and solve routine problems readily and can find ways to reach a solution or goal where no routine path is apparent.
- Communicate precisely about quantities, logical relationships, and unknown values through the use of signs, symbols, models, graphs, and mathematical terms.
- Reason mathematically by gathering data, analyzing evidence, and building arguments to support or refute hypotheses.
- Make connections among mathematical ideas and between mathematics and other disciplines.

The standards emphasize computational and procedural skills, conceptual understanding, and problem solving.

The standards identify what all students in California public schools should know and be able to do at each grade level. Nevertheless, local flexibility is maintained with these standards. Topics may be introduced and taught at one or two grade levels before mastery is expected. Decisions about how best to teach the standards and in what order they should be taught are left to teachers, schools, and school districts.

The standards emphasize computational and procedural skills, conceptual understanding, and problem solving. These three components of mathematics instruction and learning are not separate from each other; instead, they are intertwined and mutually reinforcing.

Basic, or computational and procedural, skills are those skills that all students should learn to use routinely and automatically. Students should practice basic skills sufficiently and frequently enough to commit them to memory.

Mathematics makes sense to students who have a conceptual understanding of the domain. They know not only *how* to apply skills but also *when* to apply them and *why* they should apply them. They understand the structure and logic of mathematics and use the concepts flexibly, effectively, and appropriately. In seeing the big picture and in understanding the concepts, they are in a stronger position to apply their knowledge to situations and problems they may not have encountered before and readily recognize when they have made procedural errors.

The mathematical reasoning standards are different from the other standards in that they do not represent a content domain. Mathematical reasoning is involved in all strands.

The standards do not specify how the curriculum should be delivered. Teachers may use direct instruction, explicit teaching, or knowledge-based discovery learning; investigatory, inquiry-based, problem-solving-based, guided discovery, set-theory-based, traditional, or progressive methods; or other ways in which to teach students the subject matter set forth in these standards. At the middle and high school levels, schools can use the standards with an integrated program or with the traditional course sequence of Algebra I, geometry, Algebra II, and so forth.

Schools that use these standards "enroll" students in a mathematical apprenticeship in which they practice skills, solve problems, apply mathematics to the real world, develop a capacity for abstract thinking, and ask and answer questions involving numbers or equations. Students need to know basic formulas, understand what they mean and why they work, and know when they should be applied. Students are also expected to struggle with thorny problems after learning to perform the simpler calculations on which they are based.

Teachers should guide students to think about why mathematics works in addition to how it works and should emphasize understanding of mathematical concepts as well as achievement of mathematical results. Students need to recognize that the solution to any given problem may be determined by employing more than one strategy and that the solution frequently raises new questions of its own: Does the answer make sense? Are there other, more efficient ways to arrive

at the answer? Does the answer bring up more questions? Can I answer those? What other information do I need?

Problem solving involves applying skills, understanding, and experiences to resolve new or perplexing situations. It challenges students to apply their understanding of mathematical concepts in a new or complex situation, to exercise their computational and procedural skills, and to see mathematics as a way of finding answers to some of the problems that occur outside a classroom. Students grow in their ability and persistence in problem solving by extensive experience in solving problems at a variety of levels of difficulty and at every level in their mathematical development.

Problem solving involves applying skills, understanding, and experiences to resolve new or perplexing situations.

Problem solving, therefore, is an essential part of mathematics and is subsumed in every strand and in each of the disciplines in grades eight through twelve. Problem solving is not separate from content. Rather, students learn concepts and skills in order to apply them to solve problems in and outside school. Because problem solving is distinct from a content domain, its elements are consistent across grade levels.

The problems that students solve must address important mathematics. As students progress from grade to grade, they should deal with problems that (1) require increasingly more advanced knowledge and understanding of mathematics; (2) are increasingly complex (applications and purely mathematical investigations); and (3) require increased use of inductive and deductive reasoning and proof. In addition, problems should increasingly require students to make connections among mathematical ideas within a discipline and across domains. Each year students need to solve problems from all strands, although most of the problems should relate to the mathematics that students study that year. A good problem is one that is mathematically important; specifies the problem to be solved but not the solution path; and draws on grade-level appropriate skills and conceptual understanding.

Organization of the Standards

The mathematics content standards for kindergarten through grade seven are organized by grade level and are presented in five strands: Number Sense; Algebra and Functions; Measurement and Geometry; Statistics, Data Analysis, and Probability; and Mathematical Reasoning. Focus statements indicating the increasingly complex mathematical skills that will be required of students from kindergarten through grade seven are included at the beginning of each grade level; the statements indicate the ways in which the discrete skills and concepts form a cohesive whole. [The symbol  identifies the key standards to be covered in kindergarten through grade seven.]

The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. Strands are not used for organizational purposes because the mathematics studied in grades eight through twelve falls naturally under the discipline headings algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an

integrated program. To allow local educational agencies and teachers flexibility, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The content of these disciplines must be covered, and students enrolled in these disciplines are expected to achieve the standards regardless of the sequence of the disciplines.

Mathematics Standards and Technology

As rigorous mathematics standards are implemented for all students, the appropriate role of technology in the standards must be clearly understood.

The following considerations may be used by schools and teachers to guide their decisions regarding mathematics and technology:

Technology does not replace the need for all students to learn and master basic mathematics skills.

Students require a strong foundation in basic skills. Technology does not replace the need for all students to learn and master basic mathematics skills. All students must be able to add, subtract, multiply, and divide easily without the use of calculators or other electronic tools. In addition, all students need direct work and practice with the concepts and skills underlying the rigorous content described in the *Mathematics Content Standards for California Public Schools* so that they develop an understanding of quantitative concepts and relationships. The students' use of technology must build on these skills and understandings; it is not a substitute for them.

Technology should be used to promote mathematics learning. Technology can help promote students' understanding of mathematical concepts, quantitative reasoning, and achievement when used as a tool for solving problems, testing conjectures, accessing data, and verifying solutions. When students use electronic tools, databases, programming language, and simulations, they have opportunities to extend their comprehension, reasoning, and problem-solving skills beyond what is possible with traditional print resources. For example, graphing calculators allow students to see instantly the graphs of complex functions and to explore the impact of changes. Computer-based geometry construction tools allow students to see figures in three-dimensional space and experiment with the effects of transformations. Spreadsheet programs and databases allow students to key in data and produce various graphs as well as compile statistics. Students can determine the most appropriate ways to display data and quickly and easily make and test conjectures about the impact of change on the data set. In addition, students can exchange ideas and test hypotheses with a far wider audience through the Internet. Technology may also be used to reinforce basic skills through computer-assisted instruction, tutoring systems, and drill-and-practice software.

The focus must be on mathematics content. The focus must be on learning mathematics, using technology as a tool rather than as an end in itself. Technology makes more mathematics accessible and allows one to solve mathematical problems with speed and efficiency. However, technological tools cannot be used effectively without an understanding of mathematical skills, concepts, and relationships. As students learn to use electronic tools, they must also develop the quantitative reasoning necessary to make full use of those tools. They must also

have opportunities to reinforce their estimation and mental math skills and the concept of place value so that they can quickly check their calculations for reasonableness and accuracy.

Technology is a powerful tool in mathematics. When used appropriately, technology may help students develop the skills, knowledge, and insight necessary to meet rigorous content standards in mathematics and make a successful transition to the world beyond school. The challenge for educators, parents, and policymakers is to ensure that technology supports, but is not a substitute for, the development of quantitative reasoning and problem-solving skills.[1]

[1] Complete citations for the sources following some of the mathematics problems in this chapter appear in "Works Cited" at the end of this publication.

Kindergarten Mathematics Content Standards

By the end of kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects and develop a sense of properties and patterns.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ⬭ identifies the key standards for kindergarten.

Number Sense

1.0 Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement):

1.1 Compare two or more sets of objects (up to 10 objects in each group) and identify which set is equal to, more than, or less than the other.

Are there more circles or more triangles in the following collection?

1.2 Count, recognize, represent, name, and order a number of objects (up to 30).

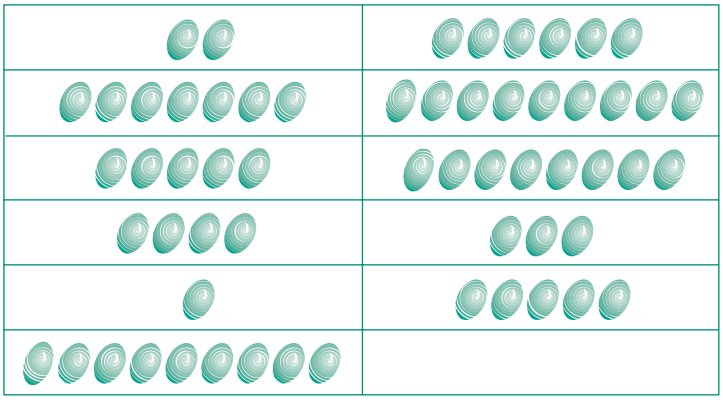
Which numbers are missing if we are counting by ones?
11, 12, 13, __, __, 16, 17, __, __, __, 21, 22, 23, 24.

1.3 Know that the larger numbers describe sets with more objects in them than the smaller numbers have.

2.0 Students understand and describe simple additions and subtractions:

2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).

Pair up as many groups of beans from the left column with groups of beans from the right column so that each group adds up to 10 beans.



3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places:

3.1 Recognize when an estimate is reasonable.

Algebra and Functions

1.0 Students sort and classify objects:

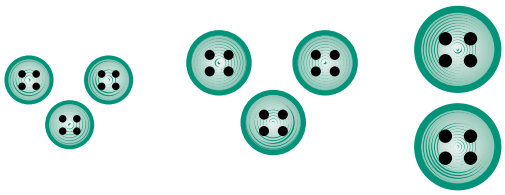
1.1 Identify, sort, and classify objects by attribute and identify objects that do not belong to a particular group (e.g., all these balls are green, those are red).

Students compare objects:

1. Which pencil is longer? Shorter?

2. Describe how the following 2 objects are the same or different.

3. Show students buttons sorted into 3 sets as shown and ask them to identify how buttons were sorted.



Measurement and Geometry

1.0 Students understand the concept of time and units to measure it; they understand that objects have properties, such as length, weight, and capacity, and that comparisons may be made by referring to those properties:

1.1 Compare the length, weight, and capacity of objects by making direct comparisons with reference objects (e.g., note which object is shorter, longer, taller, lighter, heavier, or holds more).

Who is the tallest girl in the class? The tallest boy? Which container holds more?

1.2 Demonstrate an understanding of concepts of time (e.g., morning, afternoon, evening, today, yesterday, tomorrow, week, year) and tools that measure time (e.g., clock, calendar).

If the teacher says to a class that a substitute will be teaching for the next four school days, when can the class expect their teacher to probably return? Tomorrow? Next week? Next month? Next year?

1.3 Name the days of the week.

1.4 Identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock; bedtime is 8 o'clock at night).

2.0 Students identify common objects in their environment and describe the geometric features:

2.1 Identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone).

Kindergarten

Which of these is a square?

Given 5 squares of the same size, can you make use of some or all of them to form a bigger square?

2.2 Compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners).

Statistics, Data Analysis, and Probability

1.0 Students collect information about objects and events in their environment:

1.1 Pose information questions; collect data; and record the results using objects, pictures, and picture graphs.

1.2 Identify, describe, and extend simple patterns (such as circles or triangles) by referring to their shapes, sizes, or colors.

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

1.1 Determine the approach, materials, and strategies to be used.

1.2 Use tools and strategies, such as manipulatives or sketches, to model problems.

2.0 Students solve problems in reasonable ways and justify their reasoning:

2.1 Explain the reasoning used with concrete objects and/or pictorial representations.

2.2 Make precise calculations and check the validity of the results in the context of the problem.

In a bag there are 4 apples, 3 oranges, 5 bananas, and 3 water bottles. How many pieces of fruit are in the bag altogether? How many different kinds of fruit are in the bag? How many objects altogether are in the bag?

Grade One Mathematics Content Standards

By the end of grade one, students understand and use the concept of ones and tens in the place value number system. Students add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problems.

Number Sense

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ● identifies the key standards for grade one.

1.0 Students understand and use numbers up to 100:

● **1.1** Count, read, and write whole numbers to 100.

● **1.2** Compare and order whole numbers to 100 by using the symbols for less than, equal to, or greater than (<, =, >).

Which of the following are correct and which are incorrect?

(a) 75 > 76 (b) 48 < 42 (c) 89 > 91 (d) 59 < 67 (e) 34 = 33

1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (to 20) (e.g., 8 may be represented as 4 + 4, 5 + 3, 2 + 2 + 2 + 2, 10 – 2, 11 – 3).

1.4 Count and group objects in ones and tens (e.g., three groups of 10 and 4 equals 34, or 30 + 4).

A certain brand of chewing gum has 10 pieces in each pack. If there are 14 students, what is the smallest number of packs we must buy to make sure each student gets at least one piece of gum? If there are 19 students? What about 21 students?

There are 5 quarters, 9 dimes, 3 nickels, and 8 pennies. They are supposed to be put in piles of ten (coins). How many such piles can be formed by all these coins, and how many are left over?

1.5 Identify and know the value of coins and show different combinations of coins that equal the same value.

Give each student a plastic set of 25 pennies, 5 nickels, and 2 dimes. Ask the class to find different ways to make 25 cents.

2.0 Students demonstrate the meaning of addition and subtraction and use these operations to solve problems:

● **2.1** Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory.

I had 10 cupcakes, but I ate 3 of them. How many cupcakes do I have left? How many if I had 18 and ate 5?

● **2.2** Use the inverse relationship between addition and subtraction to solve problems.

2.3 Identify one more than, one less than, 10 more than, and 10 less than a given number.

2.4 Count by 2s, 5s, and 10s to 100.

Which numbers are missing if we are counting by 2s?

24, 26, 28, 30, __, __, 36, __, 40, 42, 44, __, __, 50

Which numbers are missing if we are counting by 5s?

15, 20, 25, 30, __, __, 45, __, 55, 60, __, 70, __, 80

2.5 Show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference).

2.6 Solve addition and subtraction problems with one- and two-digit numbers (e.g., 5 + 58 = __).

If I read 16 pages on Monday, 9 pages on Tuesday, no pages on Wednesday, and 7 pages on Thursday, how many pages have I read so far this week?

2.7 Find the sum of three one-digit numbers.

3.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, and hundreds places:

3.1 Make reasonable estimates when comparing larger or smaller numbers.

Algebra and Functions

1.0 Students use number sentences with operational symbols and expressions to solve problems:

1.1 Write and solve number sentences from problem situations that express relationships involving addition and subtraction.

Do the following problems in succession:

Take away

Marie had some pencils in her desk. She put 5 more in her desk. Then she had 14. How many pencils did she have in her desk to start with?

Comparison

Eddie had 14 helium balloons. A number of them floated away. He had 5 left. How many did he lose?

Difference

1. Nina had 14 seashells. That was 5 more than Pedro had. How many seashells did Pedro have?
2. 5 + __ = 6? __ + 12 = 14?

1.2 Understand the meaning of the symbols +, –, =.

1.3 Create problem situations that might lead to given number sentences involving addition and subtraction.

Measurement and Geometry

1.0 Students use direct comparison and nonstandard units to describe the measurements of objects:

1.1 Compare the length, weight, and volume of two or more objects by using direct comparison or a nonstandard unit.

Measure your desk by using the length of a ballpoint pen. How many ballpoint pens would be roughly equal to the length of your desk? The width of your desk? Which is longer?

1.2 Tell time to the nearest half hour and relate time to events (e.g., before/after, shorter/longer).

2.0 Students identify common geometric figures, classify them by common attributes, and describe their relative position or their location in space:

2.1 Identify, describe, and compare triangles, rectangles, squares, and circles, including the faces of three-dimensional objects.

Describe the shape of a page in your textbook and compare it to the face of the clock on the wall.

2.2 Classify familiar plane and solid objects by common attributes, such as color, position, shape, size, roundness, or number of corners, and explain which attributes are being used for classification.

2.3 Give and follow directions about location.

Here are pictures on a table of a ball, a girl, a horse, and a cat. Arrange them according to these directions:

1. Put the picture of the ball above the picture of the horse.
2. Put the picture of the girl on top of the picture of the horse.
3. Put the picture of the cat under the picture of the horse.

2.4 Arrange and describe objects in space by proximity, position, and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of).

Statistics, Data Analysis, and Probability

1.0 Students organize, represent, and compare data by category on simple graphs and charts:

1.1 Sort objects and data by common attributes and describe the categories.

1.2 Represent and compare data (e.g., largest, smallest, most often, least often) by using pictures, bar graphs, tally charts, and picture graphs.

2.0 Students sort objects and create and describe patterns by numbers, shapes, sizes, rhythms, or colors:

2.1 Describe, extend, and explain ways to get to a next element in simple repeating patterns (e.g., rhythmic, numeric, color, and shape).

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

1.1 Determine the approach, materials, and strategies to be used.

1.2 Use tools, such as manipulatives or sketches, to model problems.

2.0 Students solve problems and justify their reasoning:

2.1 Explain the reasoning used and justify the procedures selected.

2.2 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students note connections between one problem and another.

Grade Two Mathematics Content Standards

By the end of grade two, students understand place value and number relationships in addition and subtraction, and they use simple concepts of multiplication. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to their geometric attributes. They collect and analyze data and verify the answers.

Number Sense

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ● identifies the key standards for grade two.

1.0 Students understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000:

● **1.1** Count, read, and write whole numbers to 1,000 and identify the place value for each digit.

1.2 Use words, models, and expanded forms (e.g., 45 = 4 tens + 5) to represent numbers (to 1,000).

Kelly has 308 stickers. How many sets of hundreds, tens, and ones does she have?

● **1.3** Order and compare whole numbers to 1,000 by using the symbols <, =, >.

Which number sentence is true? (CST released test question, 2004)[2]

(a) 359 < 375 (b) 359 > 375 (c) 359 < 359 (d) 359 > 359

2.0 Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers:

● **2.1** Understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for 8 + 6 = 14 is 14 – 6 = 8) to solve problems and check solutions.

Sophie did this subtraction problem. Which addition problem shows that she got the right answer? (CST released test question, 2004)

$$\begin{array}{r} 85 \\ -44 \\ \hline 41 \end{array}$$

(a)	(b)	(c)	(d)
41 + 85	44 + 85	41 + 44	44 + 44

[2] The Web site for accessing the *California Standards Test (CST)* released test questions for mathematics is http://cde.ca.gov/ta/tg/sr/css05rtq.asp.

2.2 Find the sum or difference of two whole numbers up to three digits long.

Use drawings of tens and ones to help find the sum 37 + 17 and the difference 25 – 19. Now do the same problems again using addition and subtraction algorithms:

$$\begin{array}{rrr} 343 & 748 & 457 \\ +\ 265 & -\ 426 & +\ 324 \end{array}$$

Grade Two

Is 37 + 118 the same as 100 + 30 + 10 + 7 + 8 ?

2.3 Use mental arithmetic to find the sum or difference of two two-digit numbers.

In a game, Mysong and Naoki are making addition problems. They make two 2-digit numbers out of the four given numbers 1, 2, 3, and 4. Each number is used exactly once. The winner is the one who makes two numbers whose sum is the largest. Mysong had 43 and 21, while Naoki had 31 and 24. Who won the game? How do you know? Show how you can beat both Mysong and Naoki by making up two numbers with a larger sum than either. (Adapted from TIMSS, gr. 3–4, V-4a)[3] (This problem also supports Mathematical Reasoning Standard 1.0.)

3.0 Students model and solve simple problems involving multiplication and division:

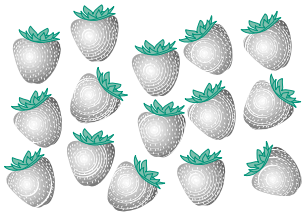
3.1 Use repeated addition, arrays, and counting by multiples to do multiplication.

Write 5×17 as a sum of numbers.

Draw a simple picture of seating 30 people in rows of 10. Show and explain how this is related to multiplication. Do this also for rows of 3, and again for rows of 5.

3.2 Use repeated subtraction, equal sharing, and forming equal groups with remainders to do division.

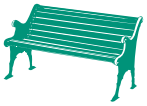
Kayla has these strawberries. She will give four strawberries to each of her three friends. How many strawberries will be left for Kayla? (CST released test question, 2004)



[3] The "Web Resources" section in "Works Cited" shows the sources in which all mathematics problems from the Third International Mathematics and Science Study (TIMSS) appearing in this publication may be found. Each problem reproduced from TIMSS is copyrighted © 1994 by IEA, The Hague.

3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.

There are nine benches in a park. There are two people sitting on each bench. How many people are sitting on the nine benches all together?
(CST released test question, 2004)

9 

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4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole:

4.1 Recognize, name, and compare unit fractions from $\frac{1}{12}$ to $\frac{1}{2}$.

True or false?

1. One-fourth of a pie is larger than one-sixth of the same pie.
2. $\frac{1}{4} > \frac{1}{3}$
3. $\frac{1}{8} < \frac{1}{10}$

4.2 Recognize fractions of a whole and parts of a group (e.g., one-fourth of a pie, two-thirds of 15 balls).

What fraction of this shape is shaded?
(CST released test question, 2004)

4.3 Know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.

Which fraction is equal to one whole?
(CST released test question, 2004)

(a) $\frac{1}{3}$ (b) $\frac{1}{8}$ (c) $\frac{2}{3}$ (d) $\frac{8}{8}$

Chapter 2
Mathematics
Content
Standards

5.0 Students model and solve problems by representing, adding, and subtracting amounts of money:

5.1 Solve problems using combinations of coins and bills.

Lee has a wallet with 5 nickels, 9 dimes, and dollar bills. In how many ways can he pay with correct change for a pen worth $1.15? What about one worth 65 cents?

Grade Two

Monique has four quarters, two dimes, and one nickel. How much money does she have? (CST released test question, 2004)

5.2 Know and use the decimal notation and the dollar and cent symbols for money.

Which of the following show a correct use of symbols for money?

(a) ¢32
(b) 72¢
(c) $1.25
(d) 2.57$

6.0 Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds, and thousands places:

6.1 Recognize when an estimate is reasonable in measurements (e.g., closest inch).

Algebra and Functions

1.0 Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction:

1.1 Use the commutative and associative rules to simplify mental calculations and to check results.

Draw pictures using dots to show:

1. Why 11 + 18 = 18 + 11.
2. Does adding 11 to 5 first and then adding the result to 17 give the same number as adding 11 to the result of adding 5 to 17?

If you know that 379 + 363 = 742, what is the sum of 363 + 379?

What number goes in the box to make this number sentence true? (CST released test question, 2004)

15 + 8 = □ + 15

1.2 Relate problem situations to number sentences involving addition and subtraction.

Andrew had 15 pennies. He found some more. Now he has 33. Which number sentence could be used to find how many pennies he found? (CST released test question, 2004)

(a) 15 + __ = 33
(b) 15 + 33 =
(c) __ – 33 = 15
(d) __ – 15 = 33

1.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.

Measurement and Geometry

1.0 Students understand that measurement is accomplished by identifying a unit of measure, iterating (repeating) that unit, and comparing it to the item to be measured:

1.1 Measure the length of objects by iterating (repeating) a nonstandard or standard unit.

1.2 Use different units to measure the same object and predict whether the measure will be greater or smaller when a different unit is used.

Four children measured the width of a room by counting how many paces it took them to cross it. It took Ana 9 paces, Erlane 8, Stephen 10, and Carlos 7. Who had the longest pace? (Adapted from TIMSS, gr. 3–4, L-8)

Measure the length of your desk with a new crayon and with a new pencil. Which is greater, the number of crayon units or the number of pencil units?

1.3 Measure the length of an object to the nearest inch and/or centimeter.

1.4 Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).

Sean is going on vacation to visit his grandparents. He will be gone one month. About how many days will Sean be gone? (CST released test question, 2004)

(a) 7 days (b) 30 days (c) 52 days (d) 365 days

Which is a longer period: 3 weeks or 19 days? 27 days or 4 weeks?

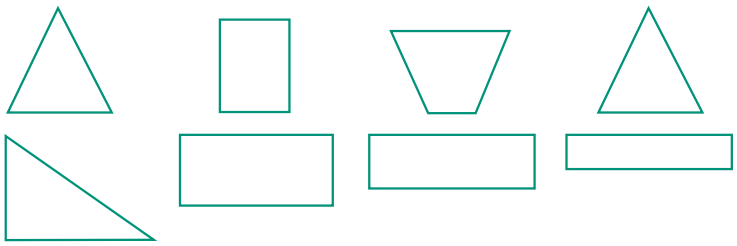
1.5 Determine the duration of intervals of time in hours (e.g., 11:00 a.m. to 4:00 p.m.).

2.0 Students identify and describe the attributes of common figures in the plane and of common objects in space:

Grade Two

2.1 Describe and classify plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid, cube, rectangular prism) according to the number and shape of faces, edges, and vertices.

Look at the pairs of shapes. Which is a pair of rectangles?
(CST released test question, 2004)



(a) (b) (c) (d)

2.2 Put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can be arranged to form a rectangle).

Statistics, Data Analysis, and Probability

1.0 Students collect numerical data and record, organize, display, and interpret the data on bar graphs and other representations:

1.1 Record numerical data in systematic ways, keeping track of what has been counted.

1.2 Represent the same data set in more than one way (e.g., bar graphs and charts with tallies).

1.3 Identify features of data sets (range and mode).

1.4 Ask and answer simple questions related to data representations.

2.0 Students demonstrate an understanding of patterns and how patterns grow and describe them in general ways:

2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 . . . ; the number of ears on one horse, two horses, three horses, four horses).

If there are two horses on a farm, how many horseshoes will we need to shoe all the horses? Show, in an organized way, how many horseshoes we will need for 3, 4, 5, 6, 7, 8, 9, and 10 horses.

2.2 Solve problems involving simple number patterns.

Mathematical Reasoning

1.0 Students make decisions about how to set up a problem:

1.1 Determine the approach, materials, and strategies to be used.

1.2 Use tools, such as manipulatives or sketches, to model problems.

2.0 Students solve problems and justify their reasoning:

2.1 Defend the reasoning used and justify the procedures selected.

2.2 Make precise calculations and check the validity of the results in the context of the problem.

3.0 Students note connections between one problem and another.

Grade Three Mathematics Content Standards

By the end of grade three, students deepen their understanding of place value and their understanding of and skill with addition, subtraction, multiplication, and division of whole numbers. Students estimate, measure, and describe objects in space. They use patterns to help solve problems. They represent number relationships and conduct simple probability experiments.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ⬬ identifies the key standards for grade three.

Number Sense

1.0 Students understand the place value of whole numbers:

1.1 Count, read, and write whole numbers to 10,000.

What is the smallest whole number you can make using the digits 4, 3, 9, and 1? Use each digit exactly once. (Adapted from TIMSS gr. 3–4, T-2)

1.2 Compare and order whole numbers to 10,000.

Which set of numbers is in order from greatest to least? (CST released test question, 2004)

(a) 147,163,234,275
(b) 275,234,163,147
(c) 275,163,234,147
(d) 163,275,234,147

⬬ 1.3 Identify the place value for each digit in numbers to 10,000.

1.4 Round off numbers to 10,000 to the nearest ten, hundred, and thousand.

Round 9,582 to the nearest thousand.

⬬ 1.5 Use expanded notation to represent numbers (e.g., 3,206 = 3,000 + 200 + 6).

Sophie has 527 seashells in her collection. Which of these equals 527? (CST released test question, 2004)

(a) 5 + 2 + 7
(b) 5 + 20 + 700
(c) 500 + 20 + 7
(d) 500 + 200 + 70

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

⬬ 2.1 Find the sum or difference of two whole numbers between 0 and 10,000.

1. 562 + 27 = ?
2. 5,286 + 2,845 = ?
3. 3,215 – 2,876 = ?

To prepare for recycling on Monday, Michael collected all the bottles in the house. He found 5 dark green ones, 8 clear ones with liquid still in them, 11 brown ones that used to hold root beer, 2 still with the cap on from his parents' cooking needs, and 4 more that were oversized. How many bottles did Michael collect? (This problem also supports Mathematical Reasoning Standard 1.1.)

2.2 Memorize to automaticity the multiplication table for numbers between 1 and 10.

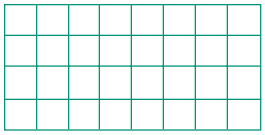
Grade Three

2.3 Use the inverse relationship of multiplication and division to compute and check results.

Use multiplication to express 24 divided by 8 = 3.

John divided 135 by 5 and got 29 as his answer. Use multiplication to see if this division problem is solved correctly.

The figure shown below is a model for the multiplication sentence $8 \times 4 = 32$.



Which division sentence is modeled by the same figure?
(CST released test question, 2004)

(a) $8 \div 4 = 2$ (b) $12 \div 4 = 3$ (c) $24 \div 8 = 3$ (d) $32 \div 8 = 4$

2.4 Solve simple problems involving multiplication of multidigit numbers by one-digit numbers (3,671 × 3 = __).

2.5 Solve division problems in which a multidigit number is evenly divided by a one-digit number (135 ÷ 5 = __).

2.6 Understand the special properties of 0 and 1 in multiplication and division.

True or false?

1. $24 \times 0 = 24$
2. $19 \div 1 = 19$
3. $63 \times 1 = 63$
4. $0 \div 0 = 1$

2.7 Determine the unit cost when given the total cost and number of units.

2.8 Solve problems that require two or more of the skills mentioned above.

A price list in a store states: pen sets, $3; magnets, $4; sticker sets, $6. How much would it cost to buy 5 pen sets, 7 magnets, and 8 sticker sets?

A tree was planted 54 years before 1961. How old is the tree in 1998?

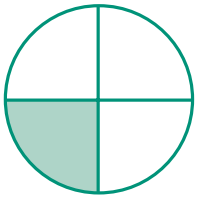
A class of 73 students go on a field trip. The school hires vans, each of which can seat a maximum of 10 students. The school policy is to seat as many students as possible in a van before using the next one. How many vans are needed?

3.0 Students understand the relationship between whole numbers, simple fractions, and decimals:

Grade Three

3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., $\frac{1}{2}$ of a pizza is the same amount as $\frac{2}{4}$ of another pizza that is the same size; show that $\frac{3}{8}$ is larger than $\frac{1}{4}$).

The circle shows $\frac{1}{4}$ shaded. (CST released test question, 2004)

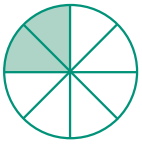


$= \frac{1}{4}$

Which fractional part of a circle below is equal to $\frac{1}{4}$?
(CST released test question, 2004)

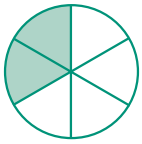
$= \frac{3}{8}$

(a)



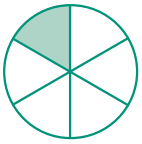
$= \frac{2}{8}$

(c)



$= \frac{2}{6}$

(b)



$= \frac{1}{6}$

(d)

3.2 Add and subtract simple fractions (e.g., determine that $\frac{1}{8} + \frac{3}{8}$ is the same as $\frac{1}{2}$).

Find the values:

1. $\frac{1}{6} + \frac{2}{6} = ?$

2. $\frac{7}{8} + \frac{3}{8} = ?$

3.3 Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors.

Pedro bought 5 pens, 2 erasers and 2 boxes of crayons. The pens cost 65 cents each, the erasers 25 cents each, and a box of crayons $1.10. The prices include tax, and Pedro paid with a ten-dollar bill. How much change did he get back?

3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is $\frac{1}{2}$ of a dollar, 75 cents is $\frac{3}{4}$ of a dollar).

Algebra and Functions

1.0 Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number relationships:

1.1 Represent relationships of quantities in the form of mathematical expressions, equations, or inequalities.

Write an inequality, equality, or expression to show each of the following relationships:

1. 12 plus a number is less than 30.
2. 4 times 6 is equal to 3 times a number.

Mr. Guzman bought 48 doughnuts packed equally into 4 boxes. Which number sentence shows how to find the number of doughnuts in each box? (CST released test question, 2004)

(a) 48 – 4 = ___ (c) 48 + 4 = ___
(b) 48 ÷ 4 = ___ (d) 48 × 4 = ___

1.2 Solve problems involving numeric equations or inequalities.

If $6 + N > 9$, circle all the numbers that N could be 3, 2, 4, 1, 0, 8, 5.

What number makes this number sentence true 3 + 5 = ___ × 2? (CST released test question, 2004)

1.3 Select appropriate operational and relational symbols to make an expression true (e.g., if 4 __ 3 = 12, what operational symbol goes in the blank?).

1.4 Express simple unit conversions in symbolic form (e.g., __ inches = __ feet × 12).

If number of feet = number of yards × 3, and number of inches = number of feet × 12, how many inches are there in 4 yards?

1.5 Recognize and use the commutative and associative properties of multiplication (e.g., if $5 \times 7 = 35$, then what is 7×5? and if $5 \times 7 \times 3 = 105$, then what is $7 \times 3 \times 5$?).

2.0 Students represent simple functional relationships:

Grade Three

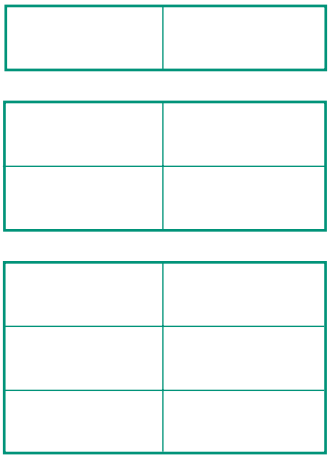
2.1 Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit).

John wants to buy a dozen pencils. One store offers pencils at 6 for $1. Another offers them at 4 for 65 cents. Yet another sells pencils at 15 cents each. Where should John purchase his pencils in order to save the most money?

One stamp costs 34¢. Two stamps cost 68¢. Three stamps cost $1.02. If the cost of each stamp remains the same, how much would 4 stamps cost? (CST released test question, 2004)

2.2 Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).

Here is the beginning of a pattern of tiles. Assuming that each figure adds two more tiles to the preceding one, how many tiles will be in the sixth figure? (Adapted from TIMSS gr. 3–4, K-6)



Measurement and Geometry

1.0 Students choose and use appropriate units and measurement tools to quantify the properties of objects:

1.1 Choose the appropriate tools and units (metric and U.S.) and estimate and measure the length, liquid volume, and weight/mass of given objects.

1.2 Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.

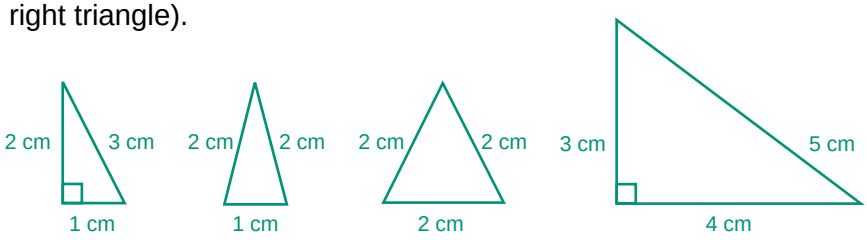
1.3 Find the perimeter of a polygon with integer sides.

1.4 Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).

2.0 Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems:

2.1 Identify, describe, and classify polygons (including pentagons, hexagons, and octagons).

2.2 Identify attributes of triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).



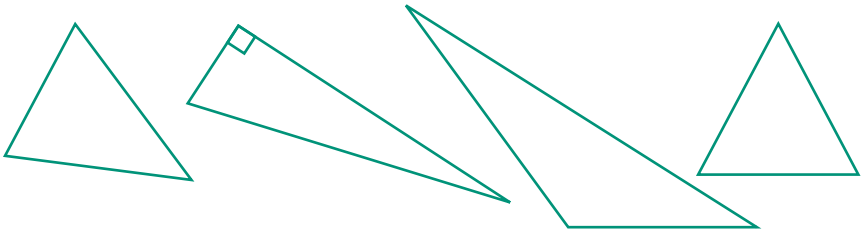
(a) (b) (c) (d)

1. Which triangle has only 2 equal sides?
2. Which triangle has 3 equal sides?
3. Which triangle(s) have one right angle?

2.3 Identify attributes of quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).

2.4 Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle.

Which of the following triangles include an angle that is greater than a right angle?



(a) (b) (c) (d)

2.5 Identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).

2.6 Identify common solid objects that are the components needed to make a more complex solid object.

Grade Three

Statistics, Data Analysis, and Probability

1.0 Students conduct simple probability experiments by determining the number of possible outcomes and make simple predictions:

1.1 Identify whether common events are certain, likely, unlikely, or improbable.

Are any of the following certain, likely, unlikely, or impossible?

1. Take two cubes each with the numbers 1, 2, 3, 4, 5, 6 written on its six faces. Throw them at random, and the sum of the numbers on the top faces is 12.
2. It snows on New Year's Day.
3. A baseball game is played somewhere in this country on any Sunday in July.
4. It is sunny in June.
5. Pick any two one-digit numbers, and their sum is 17.

1.2 Record the possible outcomes for a simple event (e.g., tossing a coin) and systematically keep track of the outcomes when the event is repeated many times.

1.3 Summarize and display the results of probability experiments in a clear and organized way (e.g., use a bar graph or a line plot).

1.4 Use the results of probability experiments to predict future events (e.g., use a line plot to predict the temperature forecast for the next day).

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

Prove or disprove a classmate's claim that 49 is more than 21 because 9 is more than 1.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.